
Recursion

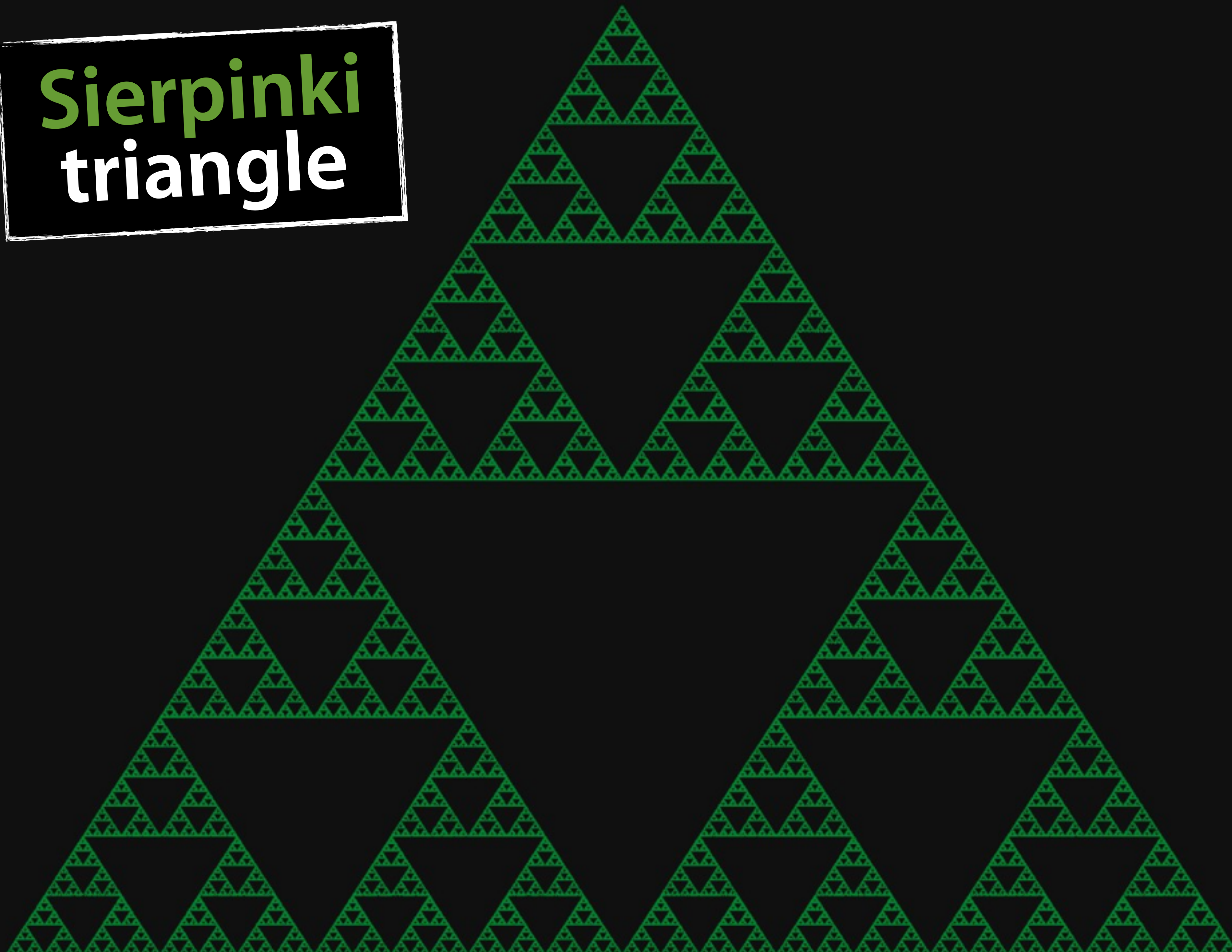
recursion

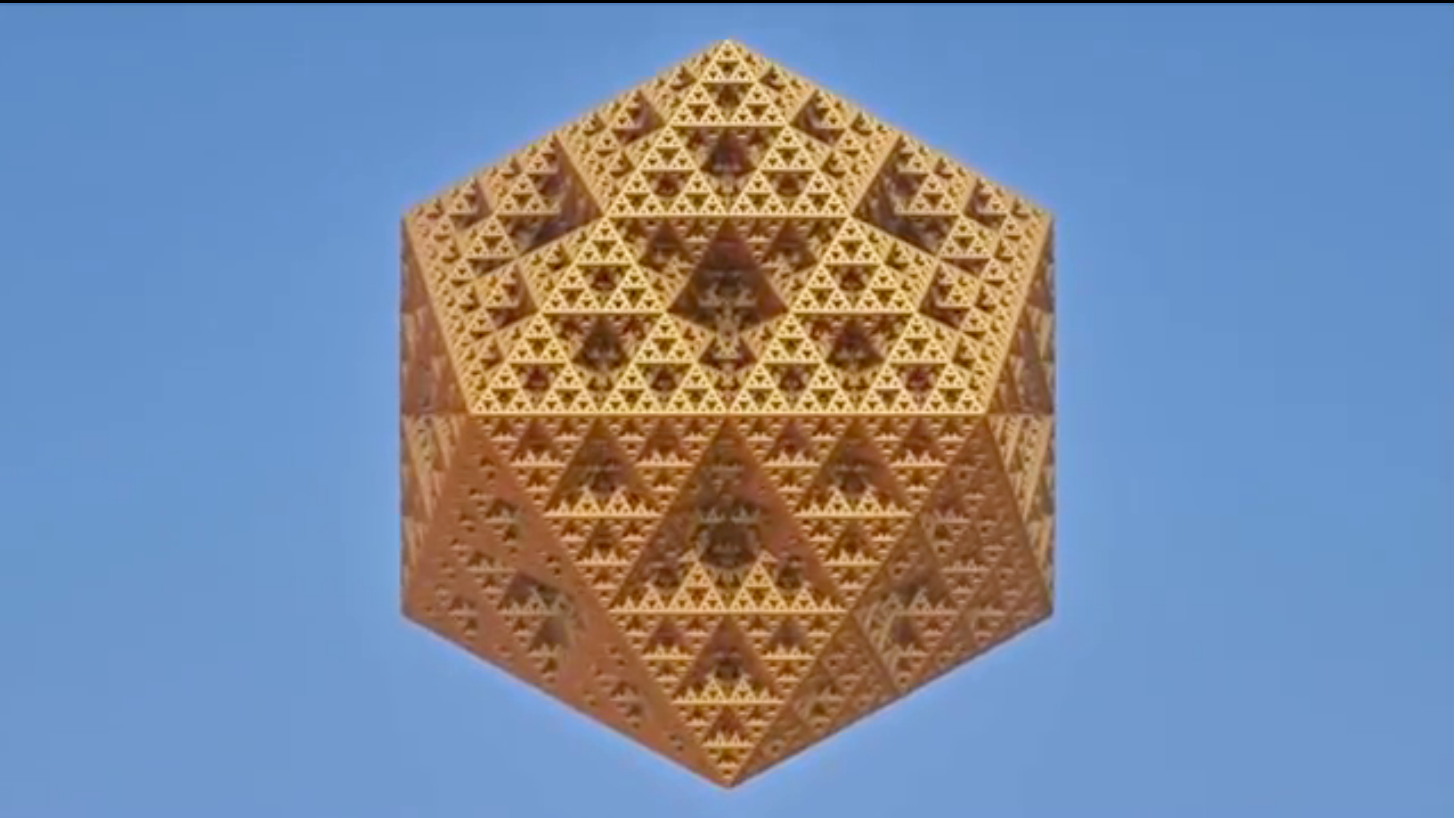
defining a function in terms of itself


DIVIDE AND CONQUER
Special Service Division
INFORMATION FILM
#3


recursive
structure

Sierpinki triangle





factorial

the product of all positive integers less than or equal to a given non-negative number

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$$5!$$

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the product of all positive integers less than or equal to a given non-negative number

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

Recursive Definition

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$$n!$$

Recursive Definition

$$n! = n \times (n-1)!$$

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$$5! = 5 \times 4!$$

Recursion Properties

Recursive solutions must satisfy three rules:

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1. **Contain a recursive case**

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1. Contain a recursive case
2. Contain a base case

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3. **Must make progress toward the base case**

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Every recursive function has an equivalent iterative solution.

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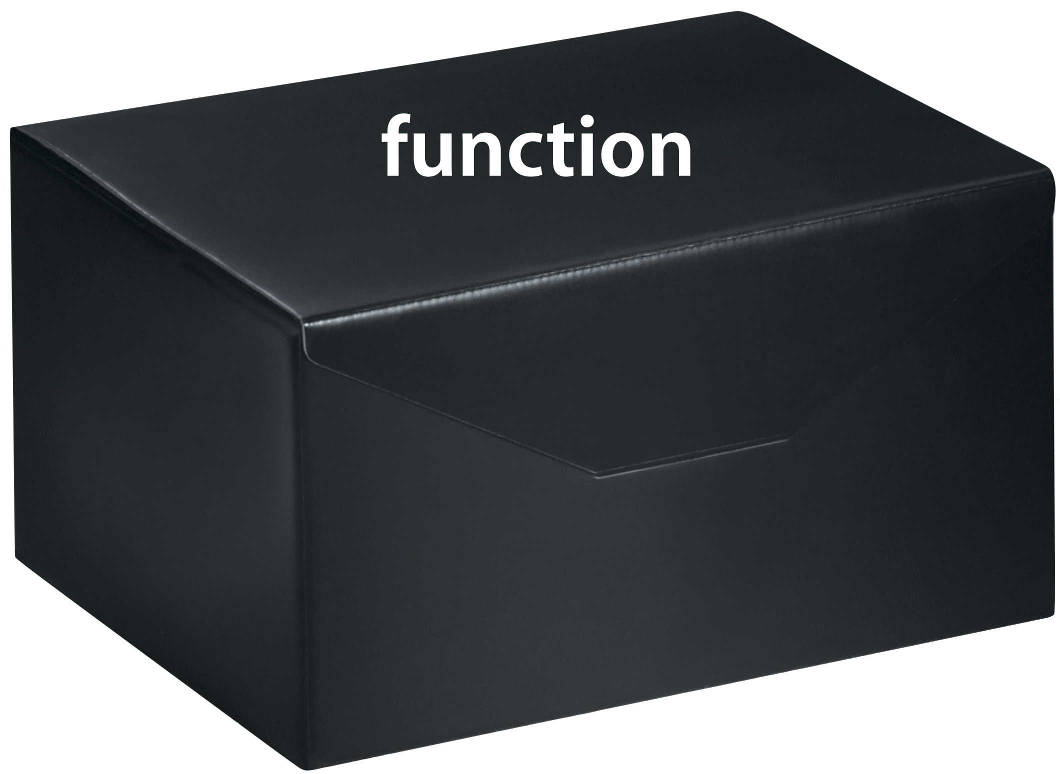
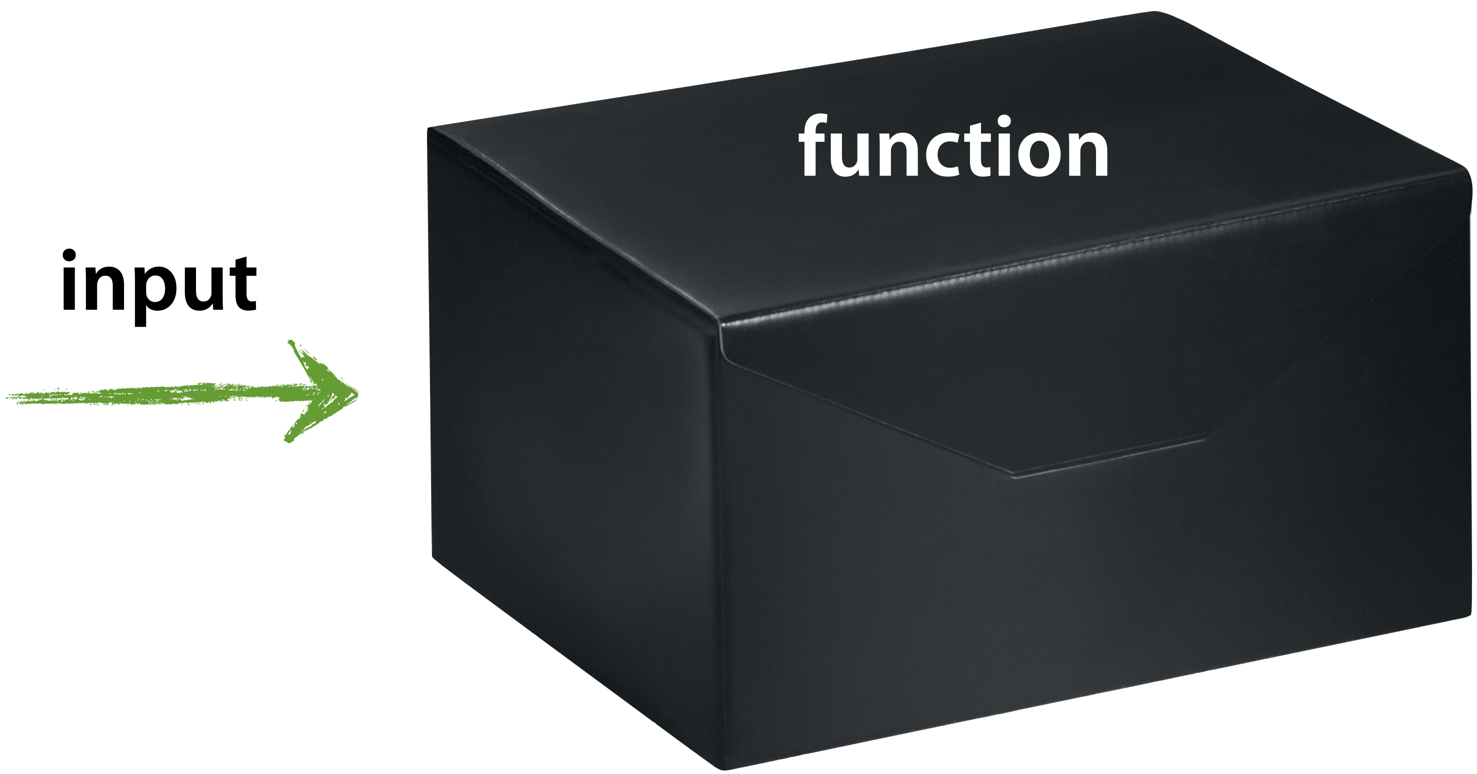
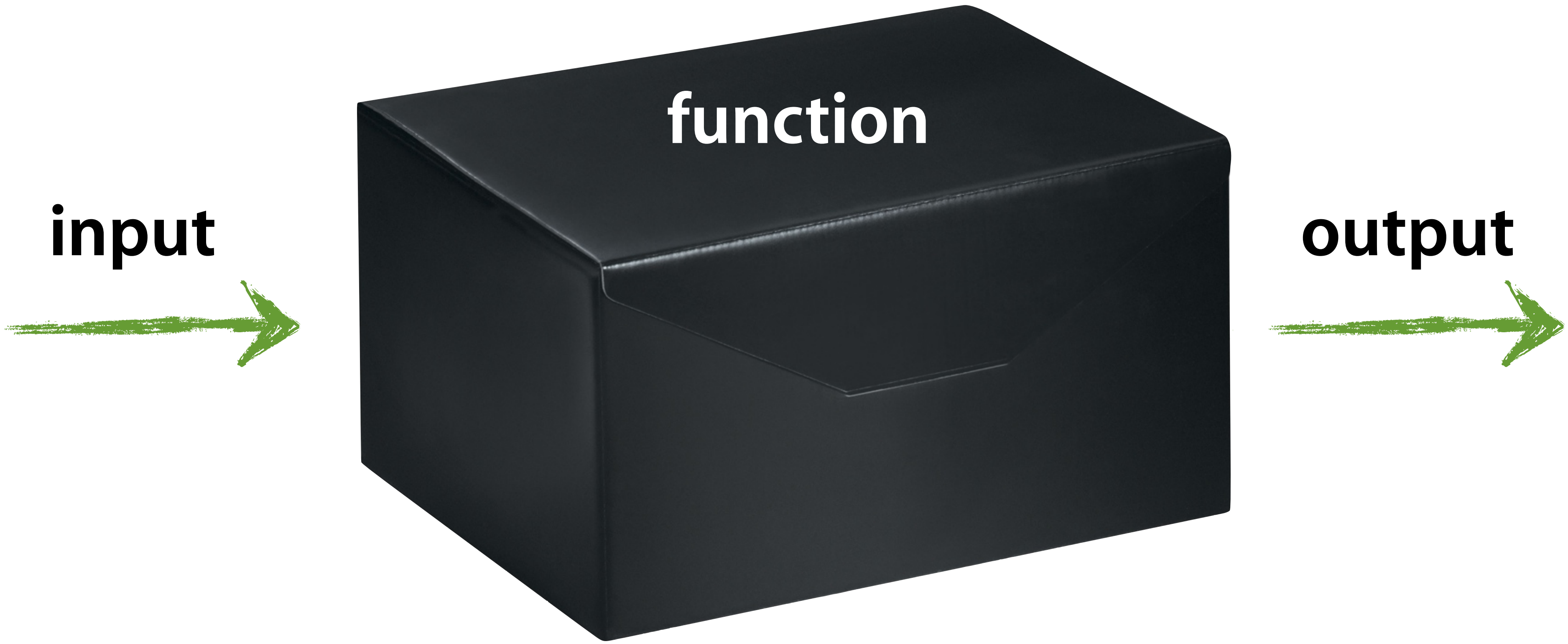
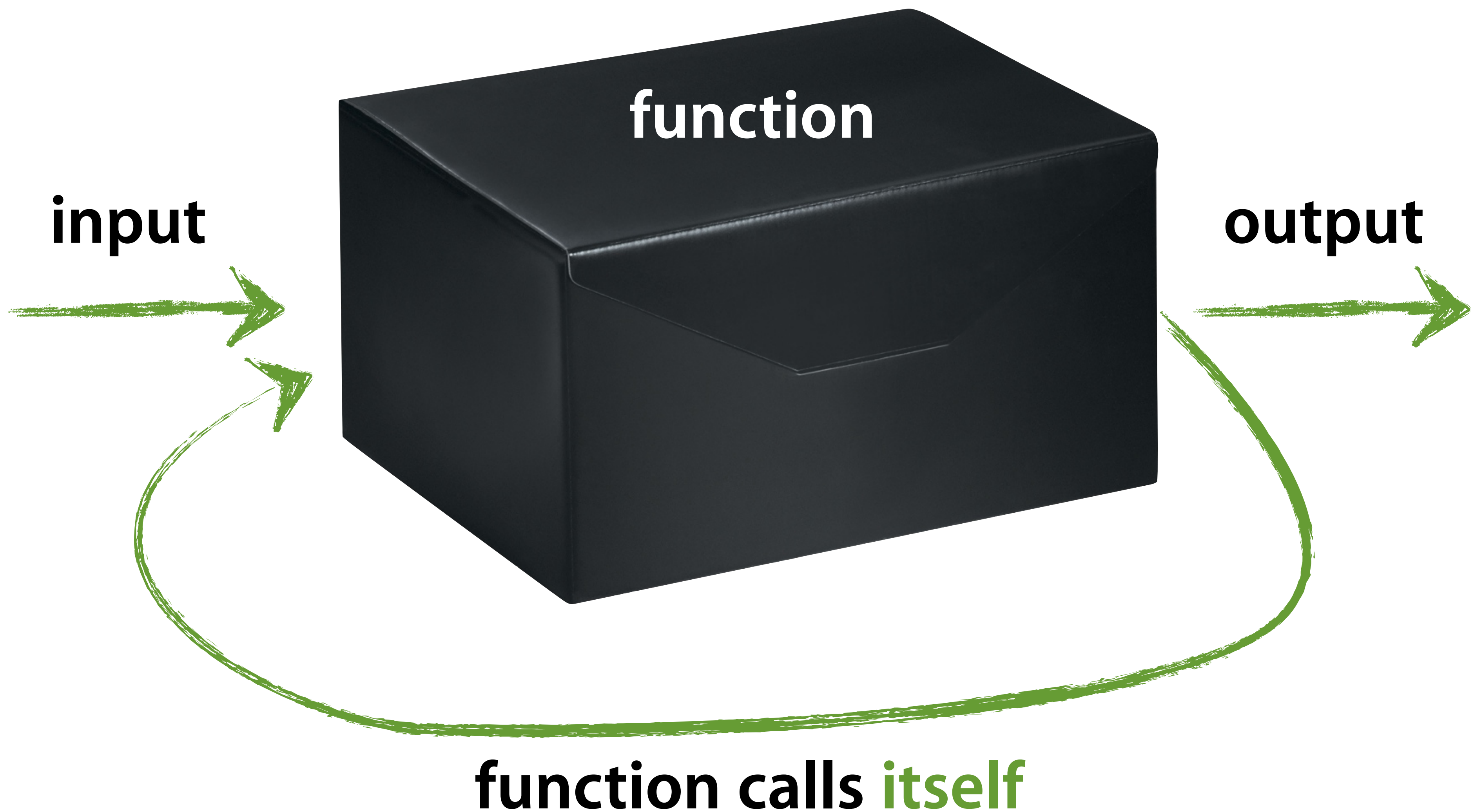
Some problems can be easier to solve one way than the other


function



input
function


input
function
output


function
input
output
function calls itself

How many students?

How many students?

iterative vs. distributed counting

Iterative Version

```
int factorial(int n)
{
    int i, product = 1;

    /* computes n*n-1... */
    for (i=n; i>1; i=i-1)
    {
        product = product * i;
    }

    /* the value returned */
    return (product);
}
```

Recursive Version

```
int factorial (int n)
{
   int product;

   if (n == 0)
      product = 1;
    else
      product = n * factorial(n-1);

   return (product);
}
```

Recursive Version

```
int factorial (int n)
{
    int product;

    if (n == 0)
        product = 1;
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What is the recursive case?

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What is the base case?

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base case

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```

Function Call Trace

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```
factorial(5)
```

factorial(5)

factorial(4)

Function
Call Trace

factorial(5)

factorial(4)

factorial(3)

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```
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    factorial(4)
        factorial(3)
            factorial(2)
```

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factorial(5)
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Function Call Trace

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factorial(5)
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                factorial(1)
                    return 1
                return 2*1 = 2
```

Function Call Trace

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factorial(5)
    factorial(4)
        factorial(3)
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                factorial(1)
                    return 1
                return 2*1 = 2
            return 3*2 = 6
```

Function Call Trace

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        factorial(3)
            factorial(2)
                factorial(1)
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                return 2*1 = 2
            return 3*2 = 6
        return 4*6 = 24
```

Function Call Trace

```
factorial(5)
    factorial(4)
        factorial(3)
            factorial(2)
                factorial(1)
                    return 1
                return 2*1 = 2
            return 3*2 = 6
        return 4*6 = 24
    return 5*24 = 120
```

Function Call Trace

```
factorial(5)
    factorial(4)
        factorial(3)
            factorial(2)
```

Computing running product starting from base case

```
                    return 1
                return 2*1 = 2
            return 3*2 = 6
        return 4*6 = 24
    return 5*24 = 120
```

Multiplication

```
int multiply(int m, int n)
{
   int answer;

   if (n == 1)
      answer = m;
   else
      answer = m + multiply(m, n - 1);

    return answer;
 }
```

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Homework

Write-out the recursive multiplication function call trace for m = 3 and n = 4.

Fibonacci number:

$$F_n = F_{n-1} + F_{n-2} \qquad n > 2$$

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$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55,\ 89,\ 144$$

Fibonacci number:

$$F_n = F_{n-1} + F_{n-2} \qquad n > 2$$

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$$F_2 = 1$$

$$F_{12}$$

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$$1, \quad 1, \quad 2, \quad 3, \quad 5, \quad 8, \quad 13, \quad 21, \quad 34, \quad 55, \quad 89, \quad 144$$

$$=$$

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$$F_1 = 1$$

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$$\begin{array}{cccccccccccc} & & & & & & & & & F_{10} & & F_{12} \\ 1, & 1, & 2, & 3, & 5, & 8, & 13, & 21, & 34, & 55, & 89, & 144 \\ & & & & & & & & & & & = \end{array}$$

Fibonacci number:

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$$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad F_{10} \qquad\quad F_{12}$$

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$$+ \qquad =$$

Fibonacci number:

$$F_n = F_{n-1} + F_{n-2} \qquad n > 2$$

$$F_1 = 1$$

$$F_2 = 1$$

$$F_{10} \quad F_{11} \quad F_{12}$$

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ 55,\ 89,\ 144$$

$$+ \quad =$$

Iterative Version

```
int fibonacci (int N)
{
    int k1, k2, k3;
    k1 = k2 = k3 = 1;

    for (int j = 3; j <= N; j++)
    {
        k3 = k1 + k2;
        k1 = k2;
        k2 = k3;
    }

    return k3;
}
```

Recursive Version

```
int fibonacci(int N)
{
   if ( (N == 1) || (N == 2) )
   {
      return 1;
   }
   else
   {
      return
       ( fibonacci(N-1) + fibonacci(N-2) );
   }
}
```

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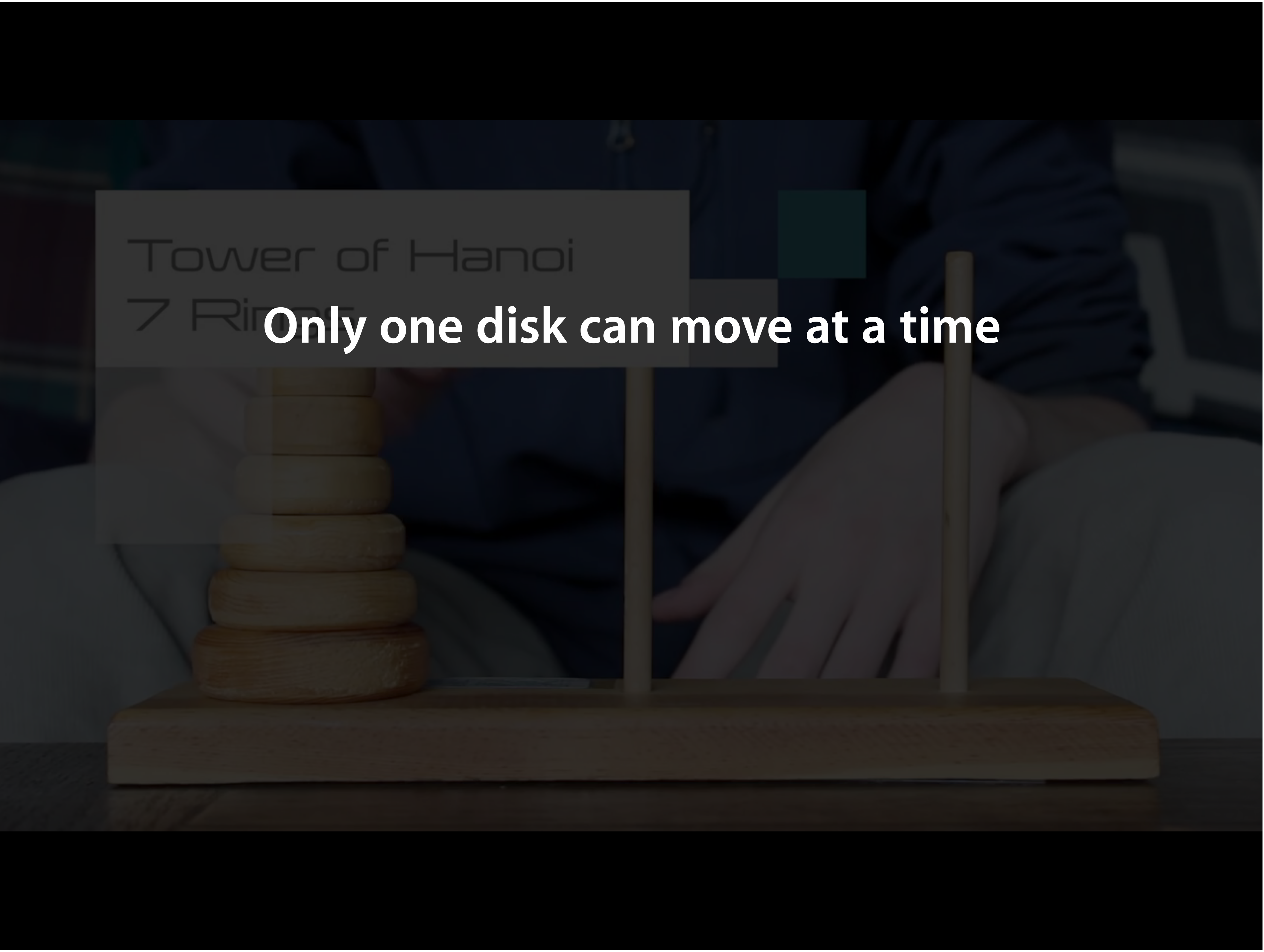
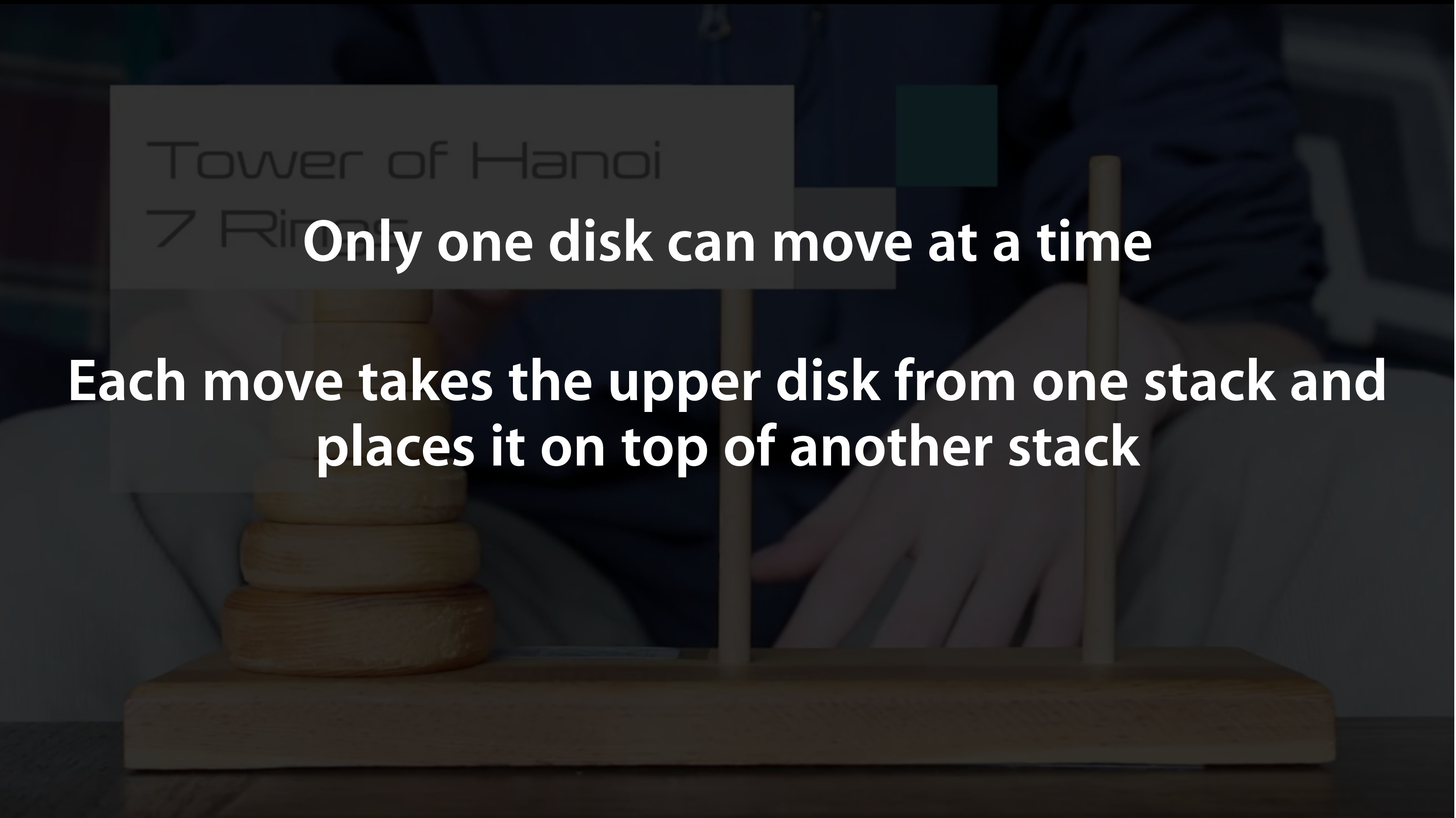
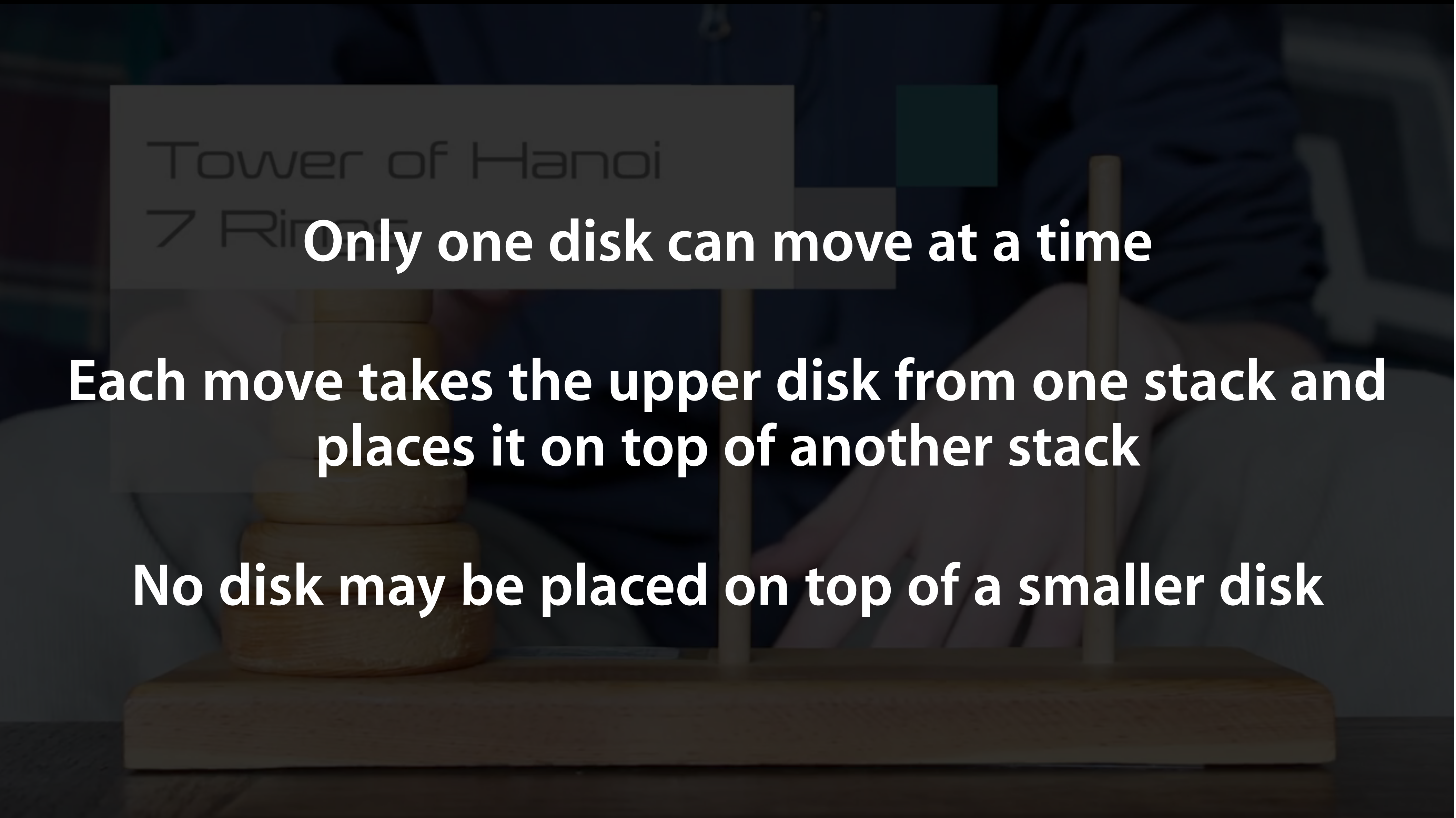

Tower of Hanoi
Only one disk can move at a time

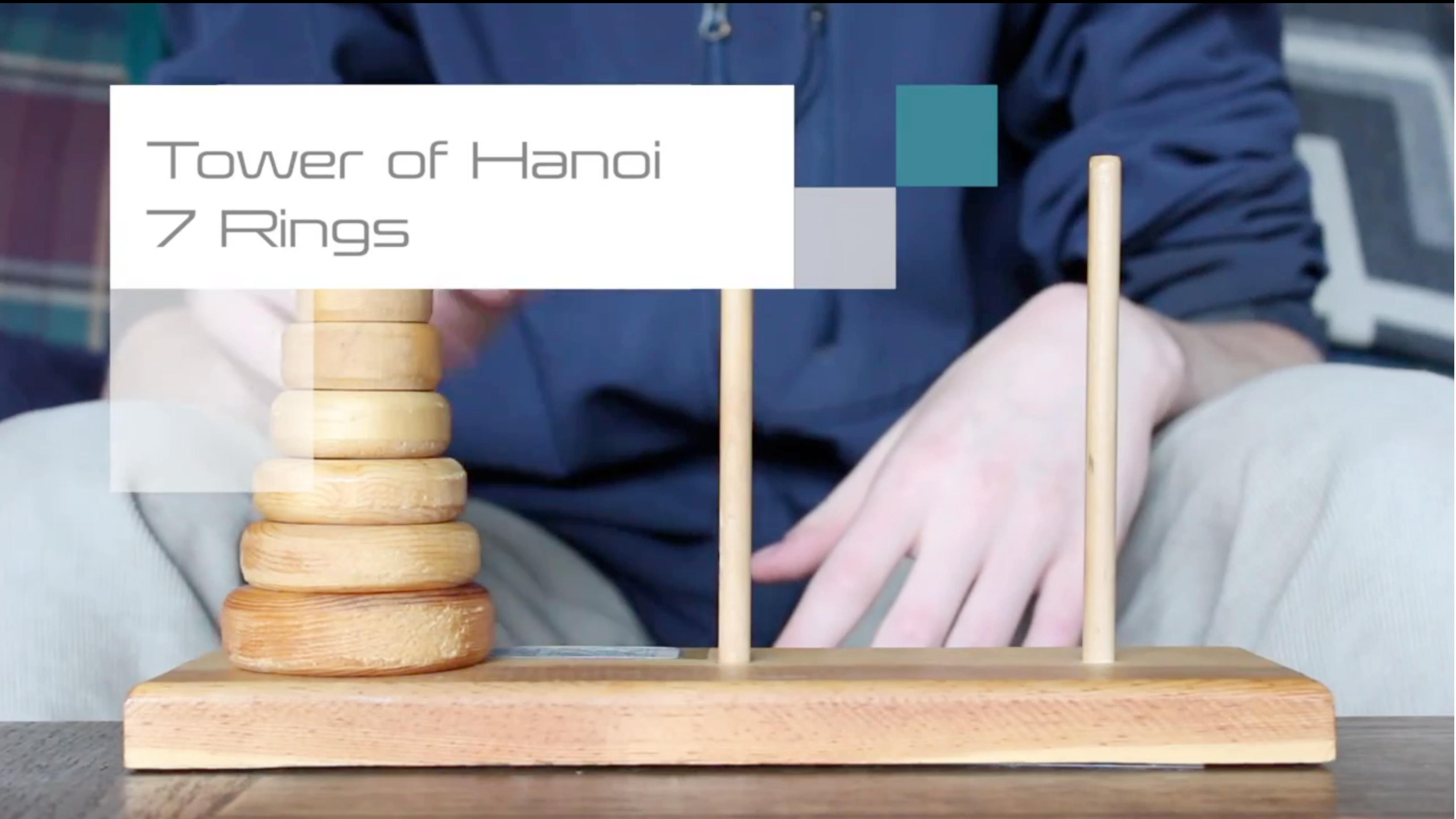


Only one disk can move at a time

Each move takes the upper disk from one stack and places it on top of another stack


Tower of Hanoi
Only one disk can move at a time
Each move takes the upper disk from one stack and places it on top of another stack
No disk may be placed on top of a smaller disk


Tower of Hanoi
7 Rings


Tower of Hanoi
7 Rings

```c
void TowersOfHanoi(int n, char a, char b, char c)
{
    if(n==1)
        printf("\nMoved from %c to %c",a,c);
    else
    {
        TowersOfHanoi(n-1,a,c,b);
        TowersOfHanoi(1,a,' ',c);
        TowersOfHanoi(n-1,b,a,c);
    }
}
```

IN ORDER TO UNDERSTAND RECURSION, ONE MUST FIRST UNDERSTAND RECURSION,


re-cur-sion (ri-kur'-zhin)
n.
1. see recursion