TORONTO METROPOLITAN UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE

CPS 420
MIDTERM
WINTER 2025

INSTRUCTIONS

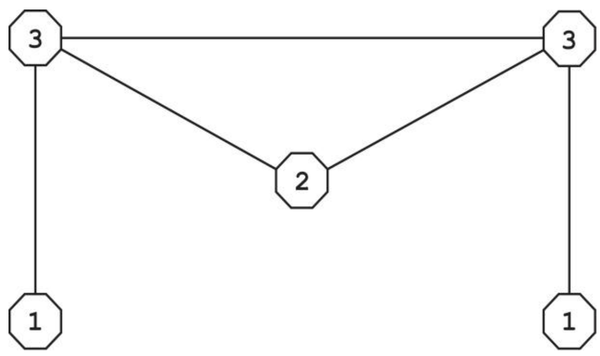
- This exam is 120 minutes long.
- This exam is out of 50 and is worth 25% of the course mark.
- This is a closed book exam. However, one double-sided letter-sized crib sheet is allowed.
- This exam is double-sided and has 9 pages including this front page. The last 2 pages are blank. Therefore, there are 6 pages of questions: pages 2 to 7 inclusive.
- Please answer all questions directly on this exam. If you need extra space to finish answering questions, please do so on pages 8 and 9 and indicate very clearly on the original page of each question on which page the rest of your answer can be found.

PART A – GRAPH THEORY – 25 MARKS

A1 Graphic Sequences (12 marks)

The *degree sequence* of a graph is the list of the degrees of all the vertices of the graph. Degree sequences are usually listed in nonincreasing order (i.e. from largest degree to smallest degree.)

A non-increasing finite sequence of non-negative integers is called a *graphic sequence* if it is the degree sequence of some simple graph (an undirected graph with no loops or parallel edges). For example, 3,3,2,1,1 is a graphic sequence because it is the degree sequence of this graph, where each vertex is labelled with its degree:



For each of the following sequences in the boxes below:

- If the sequence is graphical, draw its corresponding simple graph.
- If the sequence is not graphical, explain why this is the case. Your explanations should either refer to specific theorems or be illustrated with graphs.
- In both cases, whenever you are drawing graphs, label each vertex with its degree.

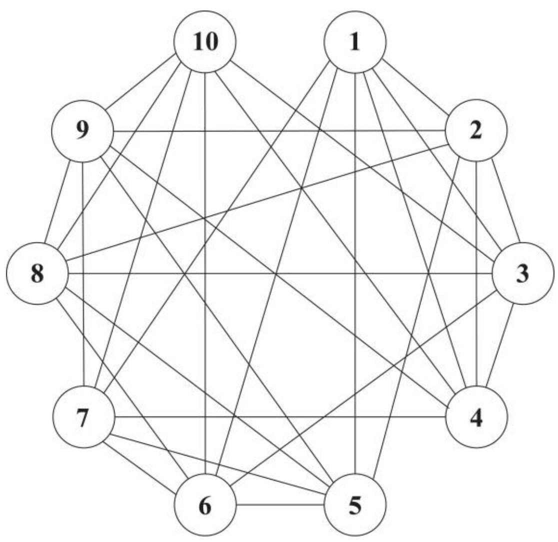
Hint: Always start by figuring out the number of vertices and edges

a) 3,3,1,1,0	b) 3,3,2,2,0

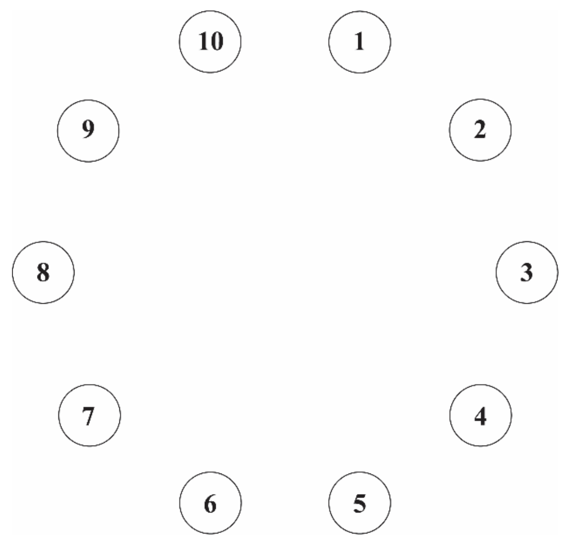
c) 4,2,1,1,1,1	d) 5,3,3,1,1,1
e) 5,3,3,2,2,1,1	f) 5,4,3,3,2,2,1

A2 Line and Petersen Graphs (13 marks)

a) (4 Marks) As you may remember from Lab5, $L(K_5)$, the *line graph* of K_5, is the graph in the left box underneath. Draw the edges of $\overline{L(K_5)}$, the complement of $L(K_5)$, in the right box.

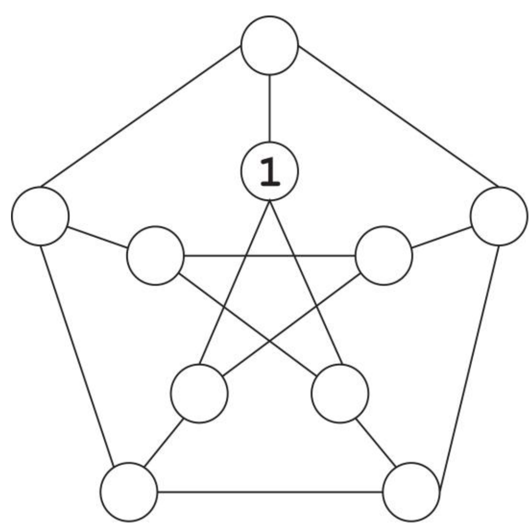


$L(K_5)$



$\overline{L(K_5)} = (L(K_5))^C$

b) (4 marks) $\overline{L(K_5)}$, is isomorphic to the *Petersen graph* drawn below in more than one way. Identify one of these isomorphisms in the graph below, assuming that the isomorphic image of vertex 1 of $\overline{L(K_5)}$, is the vertex labelled 1 in the Petersen graph. Label the remaining vertices from 2 to 10 to describe this isomorphism.



c) (5 marks) In the two boxes below you are asked whether the Petersen graph contains a specific type of trail. If the answer is yes, give the trail; if not explain why not.

Does the Peterson graph contain an **Euler circuit**?	Does the Peterson graph contain a **Hamiltonian path**?

PART B – SEQUENCES, RECURRENCE RELATIONS – 9 MARKS

Given the sequence a_n defined with the recurrence relation:

$a_0 = 0$
$a_k = 2a_{k-1} + 5k$ for $k \geq 1$

B1 Terms of the Sequence (5 marks)

Calculate a_1, a_2, a_3, a_4, a_5
Keep your intermediate answers as you will need them in the next question.

B2 Iteration (4 marks)

Guess a concise formula for a_n that uses the sum (Σ) and/or product (Π) notation.

PART C – INDUCTION – 16 MARKS

This question works with the sequence a_n defined in part B. However, you do not need to finish part B in order to be able to do this question. In particular you cannot use the answer for B2 in this proof.

Prove by **mathematical (weak) induction** that for values of n strictly greater than 1, a_n is greater or equal to $5 \cdot 2^n$.
No other method is acceptable.
Be sure to lay out your proof clearly and correctly and to justify every step.

C1 Problem Statement (3 marks)

The conjecture that you are proving, is expressed symbolically in the form ∀n∈D, P(n).

- (1.5 mark) What is the set D?

- (1.5 marks) What is the predicate function P(n)?

C2 Base Case (3 marks) Prove your base case here:

C3 Inductive step setup (4 marks)

- (2.5 marks) State the assumption in the inductive step and identify the inductive hypothesis.

- (1.5 marks) State what you will be proving in the inductive step.

C4 Remainder of Inductive step (6 marks).

Finish your proof here. Be sure to justify every step, particularly why the recursive definition can be applied and where the inductive hypothesis is applied.

THIS PAGE IS INTENTIONALLY LEFT BLANK AND CAN BE USED FOR ROUGH WORK OR TO CONTINUE ANSWERING AN EARLIER QUESTION.

WORK ON THIS PAGE WILL ONLY BE GRADED IF SPECIFICALLY REQUESTED ON ONE OF PAGES 2 TO 7.

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